TRINITY COLLEGE

Mathematics Specialist Units 1,2
Test 3 2017

Calculator Assumed

Proof, Vector Proof, Circle Geometry

STUDENT'S NAME ________________________________

DATE: Friday 19 May **TIME:** 60 minutes **MARKS**: 52

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (6 marks)

Consider the true statement:

"If quadrilateral ABCD is a rhombus, then it is a parallelogram"

(a) Write down the converse of this statement and state whether it is true or false, and if it is false, provide a counter-example. [2]

IF A PARALLELOGRAM THEN A RHOMBUS

FALSE

NOT ALL SIDES OF A PARALLELOGRAM EQUAL

(b) Write down the contrapositive of this statement and state whether it is true or false, and if it is false, provide a counter-example. [2]

IF NOT A PARALLELOGRAM THEN NOT A RHOMBUS

TRUE

(c) Write down the inverse of this statement and state whether it is true or false, and if it is false, provide a counter-example. [2]

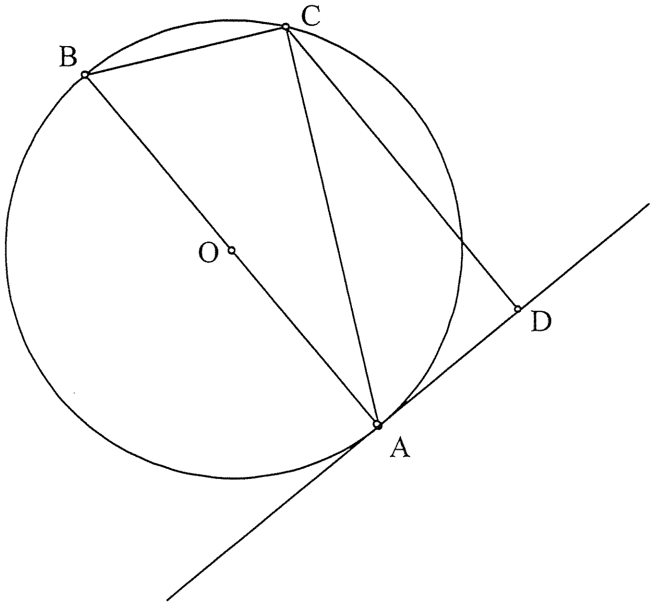
IF NOT A RHOMBUS THEN NOT A PARALLELOGRAM

FALSE

WHILE NOT A RHOMBUS IT COULD STILL BE A PARALLELOGRAM

2. (8 marks)

In this diagram, AOB is the diameter of a circle, AC is a chord of the circle and CD is perpendicular to the tangent AD.



(a) Prove ΔABC is similar to ΔCAD [3]

$\angle BCA = \angle CAB$ ALT. ANGLES

$\angle ADC = \angle ACB = 90^\circ$ GIVEN, ANGLE IN SEMICIRCLE

$\therefore \Delta ABC \sim \Delta CAD$ AA

(b) Hence show $(AC)^2 = AB.CD$ [2]

$\dfrac{ABC}{CAD}$ $\dfrac{AC}{CD} = \dfrac{AB}{AC}$

$AC^2 = AB \times CD$

(c) Determine the radius of the circle when $AC = 15\ cm$ and $AD = 12\ cm$. [3]

$CD^2 + 12^2 = 15^2$

$CD = 9$

$AC^2 = AB \times CD$

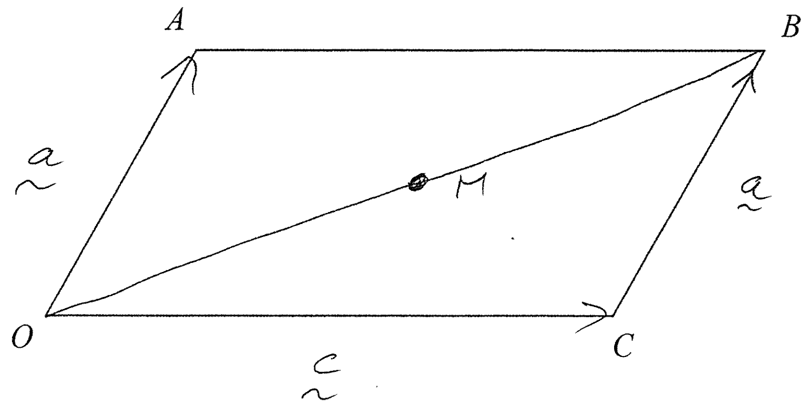
$15^2 = AB \times 9$

$\dfrac{225}{9} = AB$

$25 = AB$ $r = 12.5\ cm$

3. (6 marks)

$OABC$ is a parallelogram with $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. M is the midpoint of the diagonal OB.



(a) Determine $\overrightarrow{CM}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$ [2]

$$CM = -c + \frac{1}{2}(c+a)$$
$$= \frac{a}{2} - \frac{c}{2}$$

(b) Determine $\overrightarrow{CA}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$ [1]

$$CA = a - c$$

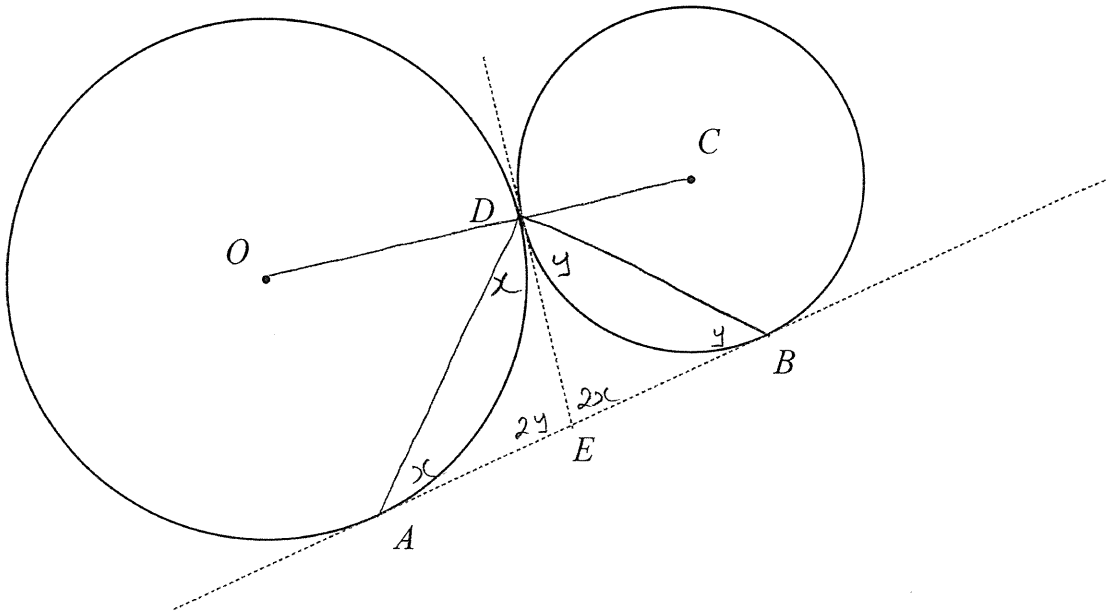
(c) Hence show that M lies on $\overrightarrow{CA}$ and is the midpoint of $\overrightarrow{CA}$. [3]

$$CM = \frac{1}{2}(a - c)$$
$$CA = a - c$$

$\therefore$ M LIES ON CA AND MIDPOINT OF CA

4. (10 marks)

The circle with centre O and the circle with centre C meet externally at D so that DE is a common tangent and AB is a tangent to both circles.



(a) Prove O, D and C are collinear. [3]

$CD \perp DE$ RADIUS $\perp$ TO TANGENT

$OD \perp DE$ RADIUS $\perp$ TO TANGENT

$\therefore$ O, D, C COLLINEAR

(b) Prove the common tangent at D bisects AB. [3]

$EB = ED$ DISTANCE FROM EXTERNAL POINT TO POINT OF TANGENCY

$EA = ED$

$\therefore$ $EB = EA$

(c) Prove $\angle ADB = 90°$. [4]

$\triangle ADE$ ISOSCELES (FROM (b))

$\angle DAE = \angle ADE = x$ $\therefore$ $\angle DEB = 2x$ EXTERNAL ANGLE

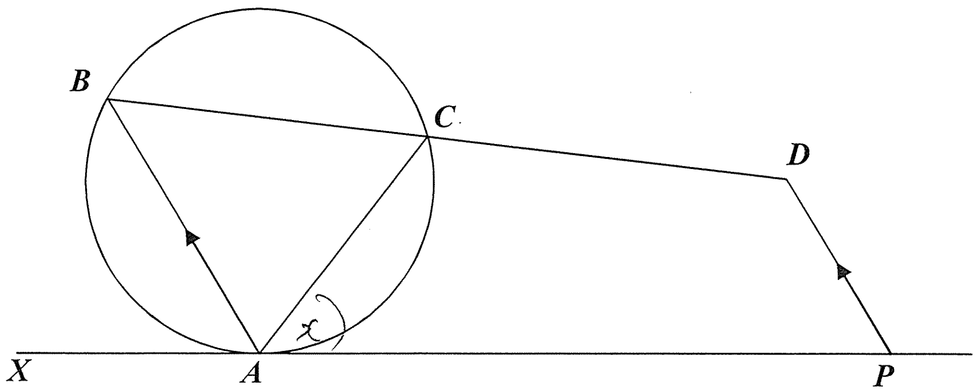
$\angle EBD = \angle EDB = y$ $\therefore$ $\angle DEA = 2y$ EXTERNAL ANGLE

$\angle AED + \angle BED = 180$ $2x + 2y = 180$

$x + y = 90 = \angle ADB$

5. (7 marks)

In the diagram, *AP* is a tangent to the circle, *D* is the point on *BC* produced such that *AB* is parallel to *PD*.



Prove that *ACDP* is a cyclic quadrilateral.

LET $\angle CAP = x$

$\angle ABC = x$ ANGLE IN ALT SEGMENT

$\angle ABC + \angle PDB = 180^\circ$ CO-INTERIOR ANGLES

$x + \angle PDB = 180^\circ$

$\angle PDB = 180 - x$

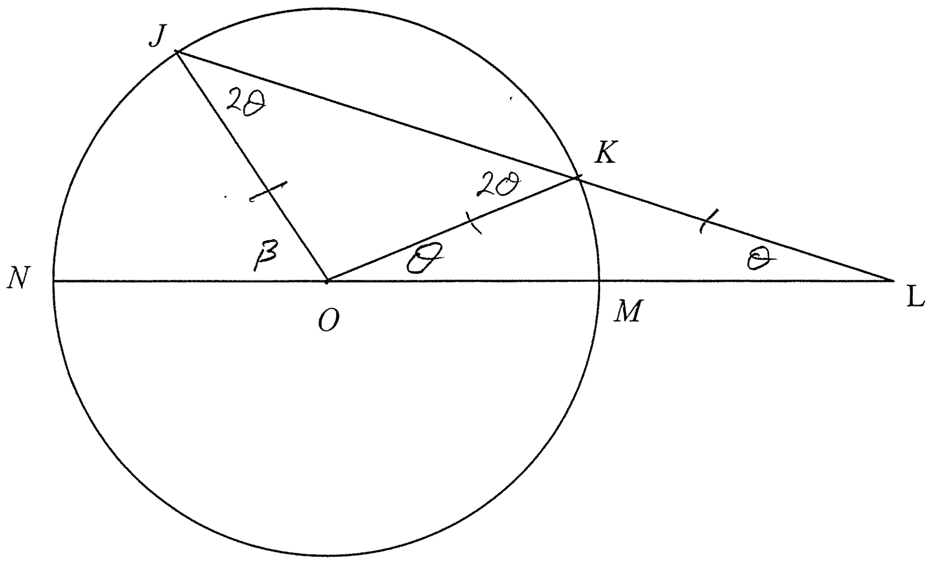
$\angle CAP + \angle PDB = x + 180 - x$

$= 180^\circ$

OPPOSITE ANGLES OF QUADRILATERAL ACDP ARE SUPPLEMENTARY

$\therefore$ ACDP CYCLIC QUAD.

6. (7 marks)



In the diagram above, O is the centre of the circle. $LMON$ and JKL are straight lines.
Let $\angle JON = \beta$ and $\angle KLM = \theta$.
The length KL is equal to the radius of the circle.

Prove that $\beta = 3\theta$.

$\angle KLM = \angle KOM = \theta$ ISOSCELES Δ

$\angle OKJ = 2\theta$ EXTERNAL ANGLE

$\angle KJO = 2\theta$ ISOSCELES Δ

$\angle JOK = 180^\circ - 2\theta - 2\theta$

$= 180 - 4\theta$

$\angle NOJ + \angle JOK + \angle KOM = 180^\circ$

$\beta + 180^\circ - 4\theta + \theta = 180^\circ$

$\beta - 3\theta = 0$

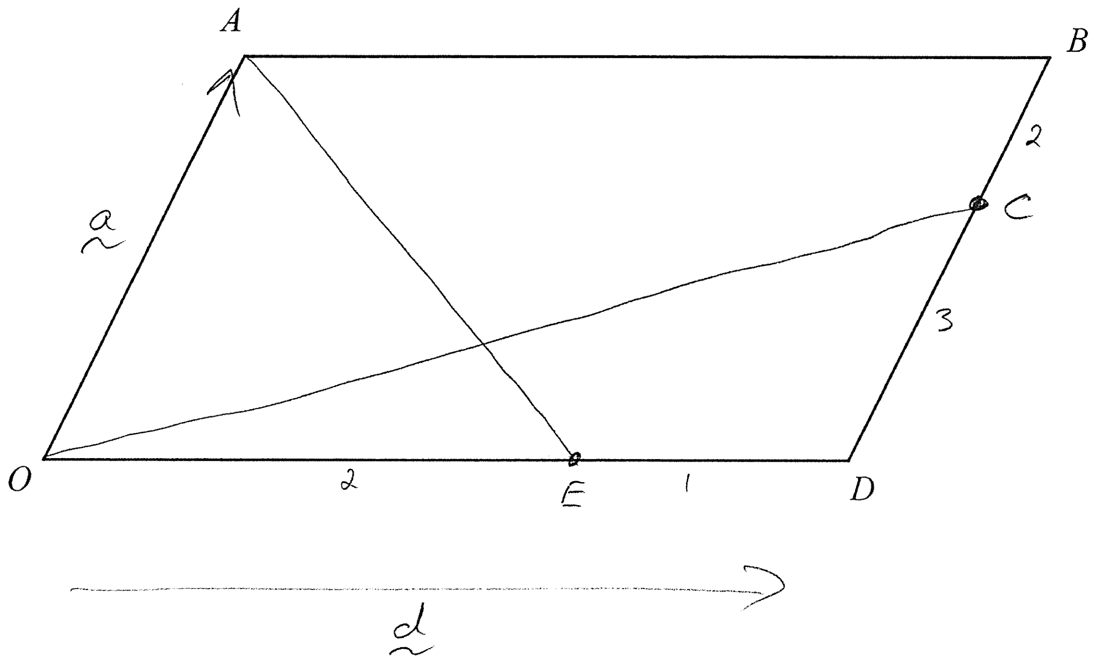
$\beta = 3\theta$

7. (8 marks)

Parallelogram OABD has C on $\overrightarrow{DB}$ such that $\overrightarrow{DC} = \frac{3}{5}\overrightarrow{DB}$ and E on $\overrightarrow{OD}$ such that $\overrightarrow{OE} = \frac{2}{3}\overrightarrow{OD}$.

Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OD} = \underset{\sim}{d}$, $\overrightarrow{OP} = h\overrightarrow{OC}$ and $\overrightarrow{AP} = k\overrightarrow{AE}$ where P is the point of intersection of $\overrightarrow{AE}$ and $\overrightarrow{OC}$.

Determine the values of h and k.



$OP = hOC$
$= h\left(d + \frac{3a}{5}\right)$

$AP = kAE$
$= k\left(\frac{2d}{3} - a\right)$

$OP = OA + AP$

$h\left(d + \frac{3a}{5}\right) = a + k\left(\frac{2d}{3} - a\right)$

$hd - \frac{3ha}{5} = a + \frac{2kd}{3} - ka$

$d\left(h - \frac{2k}{3}\right) = a\left(1 - k - \frac{3h}{5}\right)$

$h - \frac{2k}{3} = 0$

$h = \frac{2k}{3}$

$1 - k - \frac{3h}{5} = 0$

$1 - k - \frac{3 \times 2k}{5 \times 3} = 0$

$1 - k - \frac{2k}{5} = 0$

$1 = \frac{7k}{5}$

$\frac{5}{7} = k$ $\qquad h = \frac{10}{21}$